2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 10

(a) The proposition $P \cap P$ is equivalent to __________.

(b) If $A \subset B$, then $A - B =$ __________.

(c) Define partial ordering of a set.

(d) Negation of $\exists x \forall y \sim p(x, y)$ is __________.

(e) The sum of the co-efficient in the expansion of $(x + y + z)^{10}$ is __________.

(f) Write Pigeonhole principle (statement only).

(g) The equation $a_n - a_{n-1} - a_{n-2} = 0$ is a recurrence relation of order 2. (T / F)

(h) The generatimg function for the sequence

$\left\{1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!} \ldots\ldots\right\}$ is ______.

(i) Rank of a null matrix is ______.

(j) Define Kernel of a matrix.

(k) How many edges the graph $K_{3, 6}$ has ?

(l) A vertex of degree zero is called ______ vertex.

GROUP - B

2. Answer <u>any eight</u> of the following. [2 × 8

(a) Let A, B, C are three sets. Then prove that $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

(b) Show that $\lfloor x+n \rfloor = \lfloor x \rfloor + n$, for any real no. x and n.

(c) Construct truth table for $(p \wedge q) \vee ((\sim p) \rightarrow q)$.

(d) How many primes are less than 200 ? Explain your answer.

(e) How many pairs of dance partner can be selected from a group of 12 women and 20 men ?

(f) Prove that the matrix $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ is orthogonal.

(g) Prove that the determinant of an idempotent matrix is either 0 or 1.

(h) Find the eigen values of $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

(i) A graph has degree sequence 5, 5, 4, 4, 3, 3, 3, 3. How many edges does it have ?

(j) Define Hamiltonian Graphs.

GROUP - C

3. Answer any eight of the following. [3 × 8

(a) Prove that "if a^2 is an even integer, then a is an even integer" using method of contrapositive.

(b) Show that Z is countably infinite and find $|Z|$.

(c) Prove that if $ac \equiv bc \bmod(n)$ and $\gcd(c, n) = 1$ then $a \equiv b \pmod{n}$

(d) Prove that $4^n > n^4 \ \forall$ integers $n \geq 5$.

(e) Solve $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 2$, given $a_0 = -5$, $a_1 = 3$

(f) Express the matrix $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

(g) Prove that $\begin{vmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{vmatrix} = x^3(x+a+b+c+d)$.

(h) Every invertible matrix possesses a unique inverse.

(i) Prove that any graph has even number of odd vertices.

(j) Show that K_5 is not planar.

GROUP - D

*Answer **any four** questions.*

4. Prove that if A be a non-empty set and ~ be an equivalence relation on A. Let a, b ∈ A. Then the equivalence classes $\bar{a}$ and $\bar{b}$ are either equal or disjoint. [7

5. State and prove Fermat's Little Theorem. [7

6. Using mathematical Induction prove that $a^n - b^n$ is divisible by $a - b$, for $a, b \in \mathbb{Z}$ and $a - b \neq 0$, $n \geq 1$. [7

7. Reduce the following matrix into its normal form and find its Rank. [7

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

8. Determine the consistency of the given system : [7

$$x_1 + 3x_2 = 2$$

$$x_2 - 3x_4 = 3$$

$$-2x_2 + 3x_3 + 2x_4 = 1$$

$$3x_1 + 7x_4 = -5.$$

9. Prove that Isomorphic graphs have the same number of components. [7

10. What is the total number of integers with distinct digits that exceed 5500 and do not contain 0, 7, 9 ? [7